ALLOCATING HAY AND GRAIN CROPS WITH LAND, WATER, AND LABOR CONSTRAINTS

Maximizing profit and identifying governing constraints

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09/23/2020

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# Introduction

An aqueduct supplying industrial interests has an excess capacity in June, July, and August of 14,000, 18,000, and 6,000 acre-ft of water, respectively. The excess water is valuable for developing irrigated farming on nearby land. The land developer stated not more than 10,000 acres of land will be allocated to crops and irrigated using the water. Table 1 illustrates how each crop requires varying amounts of water depending on the month (Bishop, Hughes and McKee 1999, 36). This report discusses the land allocations for hay and grain crops that maximize the financial returns from the land use, as well as the governing (binding) constraints and any leftover (slack) resources in the optimal solution.

Table 1. Monthly water requirements (acre-ft) and return per acre planted

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Crop | June | July | August | Return, $/acre |
| Hay | 2 | 1 | 1 | 100 |
| Grain | 1 | 2 | 0 | 120 |

# Methods

The author utilized Microsoft Excel and the General Algebraic Modeling System (GAMS) to find the optimal solution. The model was formulated and solved using the following steps:

* Define the decision variables: the area of hay (Ah), and area of grain (Ag) to plant.
* Define the objective function describing the return from allocated crop areas (Figure 1).
* Define the constraints that govern the set of feasible crop allocations (Figure 2).
* For the Excel solution, tabulate the objective function and constraint equations and use the Excel Solver to maximize the profit by changing the decision variables under the given constraints (Figure 3). The answer and sensitivity reports (Figure 4, Figure 5) give the optimal solution and constraint information.
* For the GAMS solution, define a model using GAMS code incorporating the objective function and constraints and solve for the maximum objective function value. The code output gives the optimal solution and constraint information (Ward, HW3 2020).

# Results

Both solutions indicate the developer should plant 2,000 acres of hay and 8,000 acres of grain for a maximized return of $1.16 million. Table 2 displays the constraint outputs from both solutions. Note the GAMS output did not explicitly give slack values or classify constraints as binding like the Excel solution. Those columns were computed from leftover resources and classified after examining model outputs. Both solutions also agree on which constraints are binding and the amount of slack (leftover resources) in each non-binding constraint.

Table 2. Constraint outputs from Excel and GAMS solutions

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Constraint** | **Excel Solution** | | | | **GAMS Solution** | | | | |
| **Final Value** | **Shadow Price** | **Status** | **Slack** | **Level** | **Marginal** | **Upper** | **Slack\*** | **Binding\*** |
| Land (3) | 10000 | 80 | Binding | 0 | 10000 | 80 | 10000 | 0 | Yes |
| June Water (4) | 12000 | 0 | Not Binding | 2000 | 12000 | 0 | 14000 | 2000 | No |
| July Water (5) | 18000 | 20 | Binding | 0 | 18000 | 20 | 18000 | 0 | Yes |
| August Water (6) | 2000 | 0 | Not Binding | 4000 | 2000 | 0 | 6000 | 4000 | No |

# Conclusion

The developer should plant 2,000 acres of hay and 8,000 acres of grain for a return of $1.16 million. The profit-limiting constraints are available land area and available water in July. An additional acre of land would increase profit by $80, and an additional acre-ft of water in July would increase profit by $20. The June and August water constraints are not binding and have a surpluses of 2000 and 4000 acre-ft of water, respectively.

# References

Bishop, A Bruce, Trevor Hughes, and Mac McKee. 1999. *Water Resources Systems Analysis - Course Notes.* Logan: Utah State University. https://digitalcommons.usu.edu/ecstatic\_all/76/.

Ward, Joshua Timothy. 2020. *GRAPHICAL AND SIMPLEX SOLUTIONS TO PROBLEM 2.3: ALLOCATING CROPS WITH LAND AND WATER CONSTRAINTS.* Class Assignment, Logan: Utah State University.

—. 2020. "HW3." *JTW\_CEE5410\_Repo.* September. https://github.com/joshuatward/JTW\_CEE5410\_Repo/tree/master/HW3%20Excel\_Gams.

# Appendix A: Model Formulation

This appendix contains selected figures and tables showing the model formulation for both the Excel solution and the GAMS solution.

**Objective Function**

Maximize the return ($) produced by cultivating hay and grain

Or, for simplex solvers, minimize the negative return as shown.

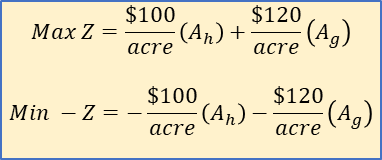


Figure 1. Objective function formulation

Table 3. Constraint definitions

|  |  |
| --- | --- |
| **Constraint** | **Definition** |
| **1** | Area of hay cannot be less than 0 acres |
| **2** | Area of grain cannot be less than 0 acres |
| **3** | Combined area of hay and grain cannot exceed 10,000 acres |
| **4** | Total water us by crops in June cannot exceed 14,000 acre-ft |
| **5** | Total water us by crops in July cannot exceed 18,000 acre-ft |
| **6** | Total water use by crops in August cannot exceed 6,000 acre-ft |

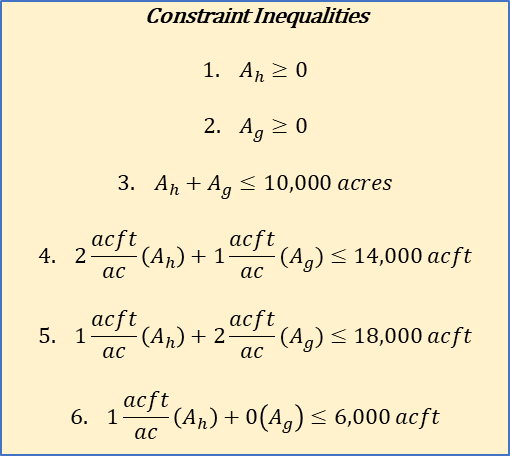


Figure 2. Mathematical formulas for problem constraints

# Appendix B: Calculations

These figures show the calculation steps and output reports for the Excel solution created by the author. The GAMS solution is stored in an [online GitHub repository](https://github.com/joshuatward/JTW_CEE5410_Repo/tree/master/HW3%20Excel_Gams) and is available to download. See the URL in the references to access the source workbook, GAMS code, and GAMS output (Ward, HW3 2020).

A screenshot of a social media post

Description automatically generated

Figure 3. Solver formulation in Excel solution



Figure 4. Answer report produced from Excel solution



Figure 5. Sensitivity report produced from Excel solution

A screenshot of a cell phone

Description automatically generated

Figure 6. Excel calculation sheet

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Category**  **(Max. Score)** | **No Evidence** | **Doesn’t Meet Standard** | **Nearly Meets Standard** | **Meets Standard** | **Exceeds Standard** | **Self- Score** | **Instructor Score** |
| **Title**  **(1)** | Absent  0 | Evidence of two or less  0 | Evidence of three  0 | Evidence of four  1 | Title – can assess main point from title alone; Name, Instructors’ Names, Course, Date, Neatly finished 1 | 1 |  |
| **Introduction**  **(3)** | Absent, no evidence  0 | There is no clear introduction or main topic.  1 | Introduction states the main topic but either:   1. Does not give a full overview, Or: 2. Too detailed, leading to annoying repetition later. 2 | The introduction states the main topic and previews the structure of the report.  2 | The introduction states the main topic and previews the structure of the report. Good overview of the problem and solution approach. Gives enough detail to motivate the reader to continue reading.  3 | 3 |  |
| **Organization and structural development of the idea: procedure, results, conclusions**  **(10)** | No content provided.  0 | Paragraphs fail to develop the main idea. No section headers or guide to help the reader understand how material is organized.  1 – 5 | Organization of ideas not fully developed. Paragraphs lack supporting detail sentences. No transitions and/or ineffective section headers.  6 - 7 | Paragraph development present but not perfected. Each paragraph has sufficient supporting detail sentences. Few transitions.  8 | Writer demonstrates logic and sequencing of ideas through well-developed section headers, paragraphs, and transitions. The first sentence of each paragraph is the summary sentence.  9 - 10 | 10 |  |
| **Technical Correctness**  **(70)** | Questions not addressed.  3 – 42% | The writer has no clue what they are talking about.  45 – 58% | Sketchy: left out required design points. Did not work on this as much as you should have, and it shows. Many important answers are incorrect.  61 – 79% | Discussion lacks adequate detail, but all the necessary points are covered and nearly all answers are correct.  82 – 88% | Provides what was explicitly asked for. The function of each piece is demonstrated to the reader in adequate, but not overwhelming, detail. Answers are correct and reasonable.  91 – 100% |  |  |
| 1. LP Model Formulation (5) | | | | | 5 |  |
| 1. Excel Solution (25) | | | | | 25 |  |
| 1. Description of model set up (10) | | | | | 10 |  |
| 1. Objective function and variable values (10) | | | | | 5 |  |
| 1. Binding constraints and non-binding slack values (5) | | | | | 5 |  |
| 1. GAMS Solution (25) | | | | | 25 |  |
| 1. Description of model set up (10) | | | | | 10 |  |
| 1. Objective function and variable values (10) | | | | | 10 |  |
| 1. Binding constraints and non-binding slack values (5) | | | | | 5 |  |
| 1. Compare Excel and GAMS software and solutions (15) | | | | | 13 |  |
| **Category**  **(Max. Score)** | **No Evidence** | **Doesn’t Meet Standard** | **Nearly Meets Standard** | **Meets Standard** | **Exceeds Standard** | **Self- Score** | **Instructor Score** |
| **Word Usage and Format**  **(10)** | Not applicable | Numerous and distracting errors in punctuation, capitalization, spelling, sentence structure, word usage, significant figures, tables, and figures. Data vomited onto page(s). Unacceptable / unprofessional at the graduate level. 1 – 5 | Misspelled words, poor English grammar and word choice. Main body of report is either longer or significantly less than one page. Figures are too small and/or under-labeled, although they are usually of acceptable quality and focus. Tables incoherent or not cohesive. Bad font sizes. Too much or too little data in appendices. Could be improved by being more meticulous.  6 - 7 | Almost no errors in punctuation, capitalization, spelling, sentence structure, word usage, significant figures, and presentation of figures, tables, and appendices. Main body of report is one page or less  8 | Punctuation, capitalization, spelling, sentence structure, word usage, and significant figures all correct. Main body of report is one page or less. Clear, consistent fonts. Good word processing skills. Figures have adequate contrast. Informative figure and table titles and legends. Figures have appropriate axis tick spacing, labels, units, and legends. Table columns cohesive, labeled, and specify units. Document is stapled. Appendices, if provided, are separated by topic, and each have a title, discussion, and proper formatting and display of information 9 - 10 | 10 |  |
| **Conclusion**  **(4)** | Absent  0 | Incomplete and/or not focused. 1 | The conclusion does not adequately restate the main results. 2 | The conclusion restates the main results. 3 | The conclusion restates the main results, and is an effective summary. 4 | 4 |  |
| **References**  **(0)** | Absent  0 | Numerous errors, off-the-wall sources used. 0 | Some errors in citing format; more sources should be cited.  1 | Prior work cited with few errors.  2 | All prior work and data sources are cited in the correct format with no errors.  2 | 0 |  |
| **TOTAL** (98) |  | | | | | 96 |  |